An Introduction to Markov Chain Models: Transitions between Robustness, Frailty, and ADL-Dependence in Late Life

E. Guilley*, A. Berchtold**, F. Armi*, & P. Ghisletta*
*Center for Interdisciplinary Gerontology, Uni Geneva
**Institute of Applied Mathematics, Uni Lausanne

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Objectives

1. To document health transitions or health trajectories with Markov Chain (MC) and Double Chain Markov Model (DCMM)

2. To compare MC models with regressions

Participants to Swilsoo

(Swiss Interdisciplinary Longitudinal Study on the Oldest-Old)

- Primary Investigator: Prof. Christian Lalive d'Epinay
- Supported by the Swiss National Science Foundation

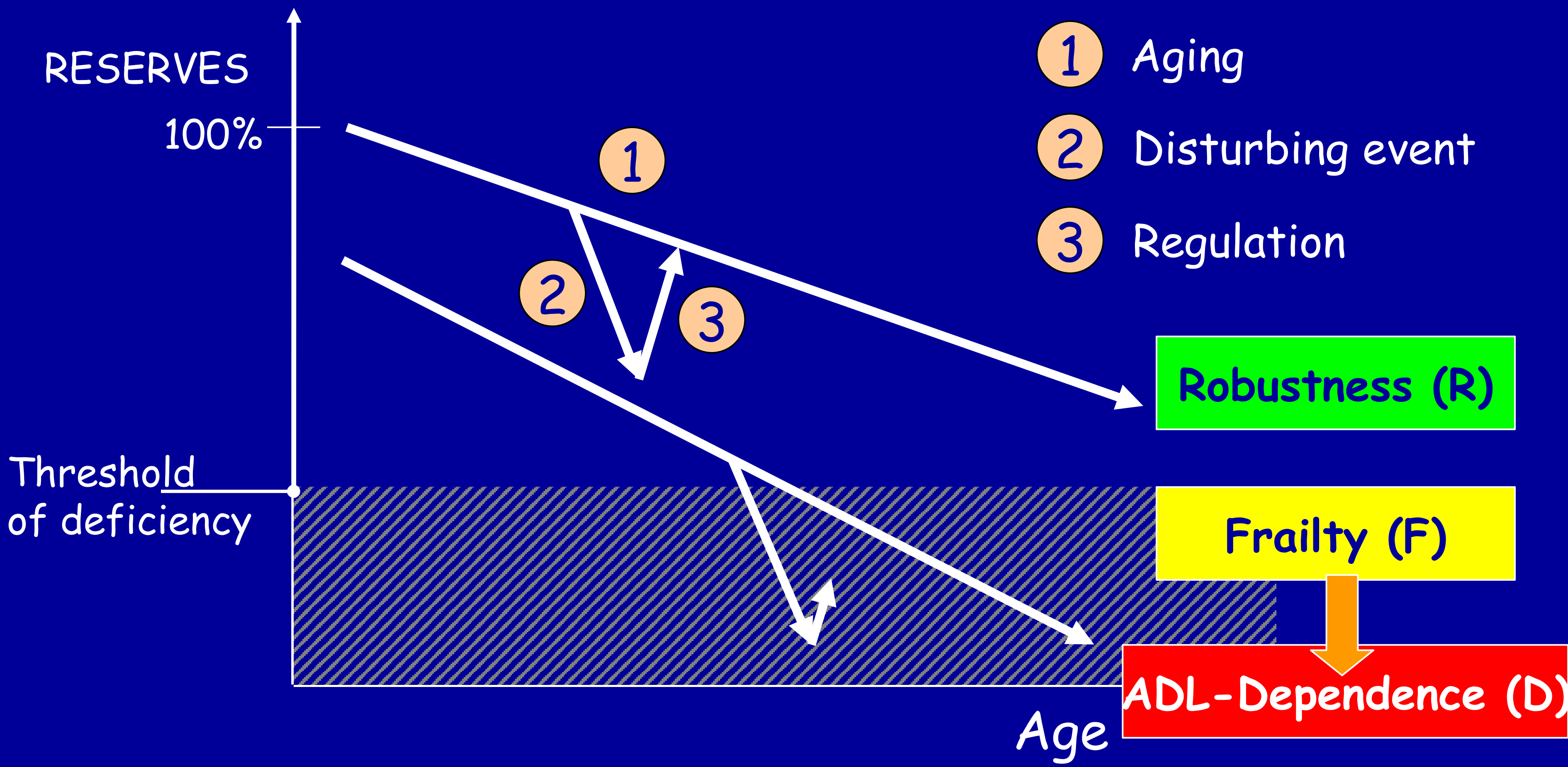
		1994	1995	1996	1997	1999	2000	2001	2002	2004
1st cohort	N	340	267	237	209	172	128	100	80	59
	Mean age	81.9	83.2	84.3	85.2	86.6	88.0	88.9	89.8	91.4
2nd cohort	N					377	289	247	212	173
	Mean age					81.9	83.2	84.2	85.1	86.6

1995–1996: 12 months

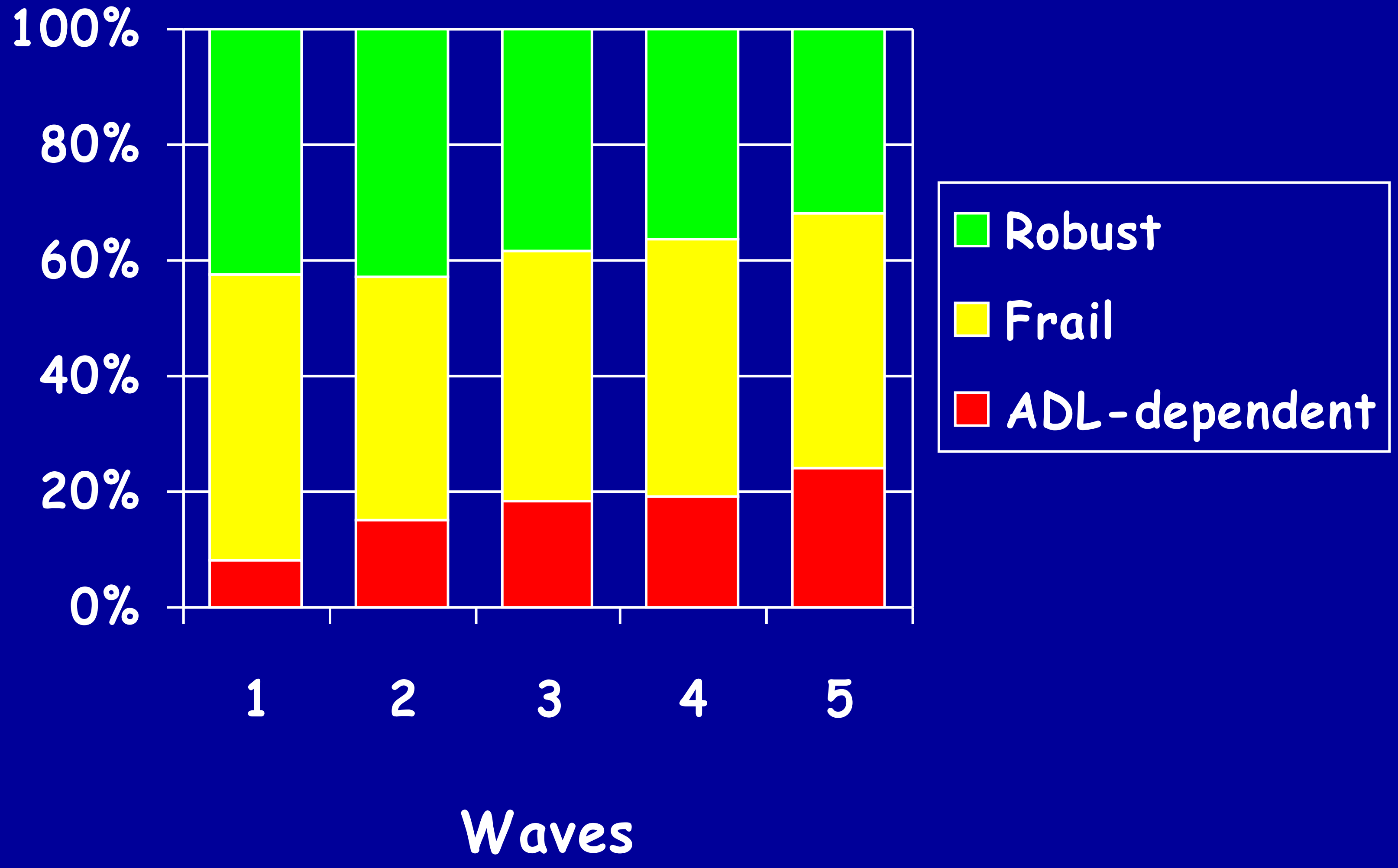
1999–2000: 18 months

- Residing at home at study inception
- Sample stratified by gender (~50% F) and geographical area (~50% urban and ~50% semi-rural)

Health states



5-wave evolution of health status for the two cohorts



Contingency-table
Homogeneous 1st order MC

… F – F – F – D …

… R – R – R – F – F – F …

F – D – D – ✝

12 or 18 months

4 destination states

T-1 \ T	R	F	D	✝	Total
R	558	257	43	41	899
F	195	639	142	112	1088
D	14	49	248	109	420
					2407

3 states of origin

Transition matrix - 1st order Markov Chain

12 or 18 months

T-1 \ T	R	F	D	†	Total
R	0.62	0.29	0.05	0.04	1
F	0.18	0.59	0.13	0.10	1
D	0.03	0.12	0.59	0.26	1

Transition matrix
-
1st order Markov Chain

12 or 18 months

4 destination states

T-1 \ T	R	F	D	✝	Precision
R	0.62	0.29	0.05	0.04	± 0.04
F	0.18	0.59	0.13	0.10	± 0.03
D	0.03	0.12	0.59	0.26	± 0.06

3 states of origin

Transition matrix on 3 successive waves: 2nd order Markov Chain

4 destination states

T-2	T-1 \ T	R	F	D	✞	Total
R	R	0.69	0.23	0.03	0.04	1
R	F	0.29	0.51	0.10	0.10	1
R	D	0.17	0.14	0.53	0.17	1
F	R	0.37	0.52	0.05	0.06	1
.	.					
.	.					
.	.					

9 trajectories of origin

Continued…

Double Chain Markov Model (DCMM)

MC

A unique transition matrix used for the whole sample

(Homogeneous)

DCMM

Different transition matrices C_1, C_2, ...

can be used to model different portions of the observed data

(Non-homogeneous)

Double Chain Markov Model (DCMM) with SWILSO-O

Two transition matrices
C_1 and C_2

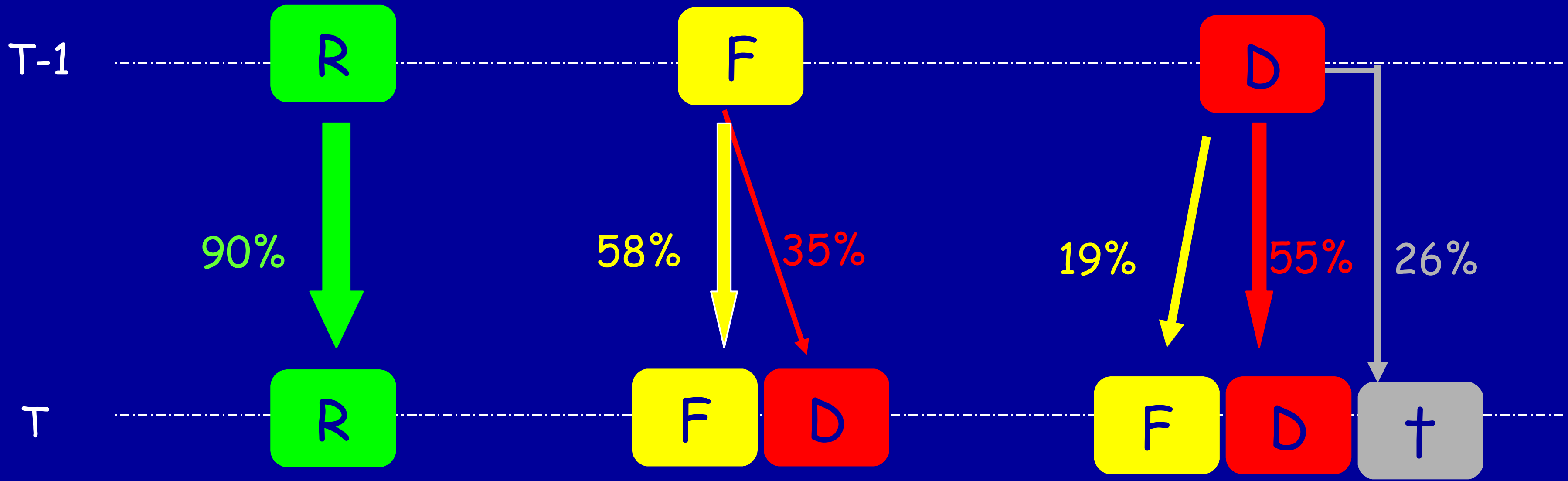
➡ This DCMM helps finding typologies of trajectories

Double Chain Markov Model (DCMM)

Typologies of trajectory

C_1

T-1 \ T	R	F	D	†
R	0.90	0.06	0.00	0.04
F	0.00	0.58	0.35	0.07
D	0.00	0.19	0.55	0.26



Double Chain Markov Model (DCMM)

Typologies of trajectory

C_2

T-1 \ T	R	F	D	✝
R	0.50	0.38	0.07	0.05
F	0.23	0.59	0.07	0.11
D	0.06	0.07	0.62	0.26

MC and DCMM can be computed using MARCH available at http://www.andreberchtold.com

Markov Chains versus Regressions: differences

Markov Chains	**Regressions**
Non-parametric; No assumption	Parametric; Assumptions should be verified at posteriori
No error terms	Estimation of the probability of Y_t
Description of every possible transitions	Limited number of states for Y_t (multivariate, ordinal)

Markov Chains versus Regressions Similarities

	Markov Chains	Regressions
Temporal homogeneity or non-homogeneity	Comparison between homogeneous and non-homogeneous MCs with BIC	Interaction between the probability of transition and Time $Y_t=f(Y_{t-1}, \text{wave}, Y_{t-1}*\text{wave})$
Time dependence of Y_t	Comparison between first-order and second-order homogeneous MC	Lag as covariables $Y_t=f(Y_{t-1}, Y_{t-2})$
Covariables	Yes – Mixture Transition Distribution	Yes

Conclusions

- MC: alternative when the assumptions of regressions are not acceptable
- MC: describes all possible transitions for exploratory analyses
- DCMM: helps finding typologies of trajectories

Thank You

edith.guilley@cig.unige.ch

This presentation can be found at:
http://www.unige.ch/centres/cig/

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References

Berchtold A., Sackett, G. (2002) Markovian Models for the Developmental Study of Social Behavior, Am. J. of Primatology 58:149-167

Brémaud (1999) Markov Chains, Springer

Ross (2002) Introduction to Probability Models, 8th edition, Academic Press

Double Chain Markov Model (DCMM)

Matrix A

T-1 \ T	C_1	C_2
C_1	1	0
C_2	0	1

$$BIC = -2\,LL + p \times \log(n)$$

DCMM:

computed by maximization of the log-likelihood using an EM algorithm. It reestimates successively the A and C transition matrices of the model until convergence, with the highest probability of generating the data.

Test of covariables
1st order MC

Men

T-1 \ T	R	F	D	✝
R	0.63	0.28	0.04	0.05
F	0.21	0.53	0.16	0.10
D	0.05	0.11	0.53	0.32

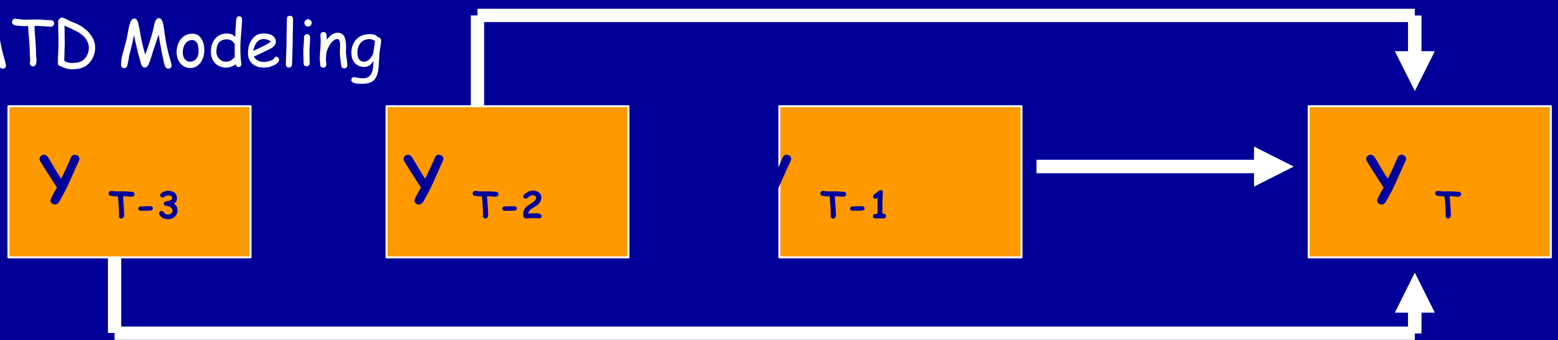
Women

T-1 \ T	R	F	D	✝
R	0.61	0.34	0.03	0.02
F	0.13	0.64	0.16	0.08
D	0.01	0.16	0.66	0.16

Mixture Transition Distribution model

3rd order Markov Chain

MTD Modeling



Using theoretically defined models: Hypothesis testing

$$BIC = -2\ LL + p \times \log(n)$$

BIC (1) with the « real » data
BIC (2) with a theoretical model